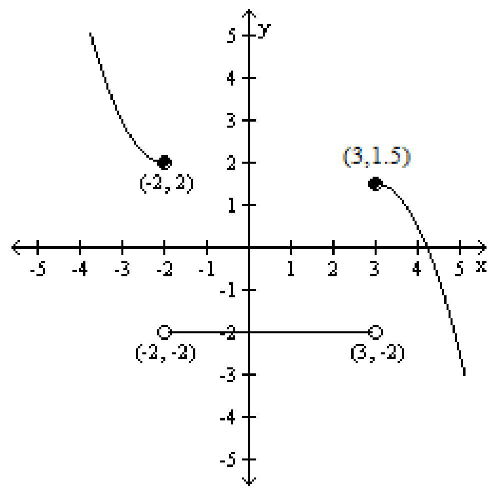
MTH 170 (College Algebra/Pre-Calc A) Final Exam Review

Find the equation of the line (in slope-intercept form) satisfying the given conditions.

1. Passing through $\left(\frac{3}{4}, \frac{1}{4}\right)$ and perpendicular to the line passing through $(-3, -5)$ and $(-4, 0)$.
2. Passing through $(5, -4)$ and parallel to $3x - 5y + 7 = 0$.

Use the graph to answer the following questions.

3. Determine the intervals on which the following function is (a) increasing, (b) decreasing, and (c) constant. Then give the (d) domain and (e) range.

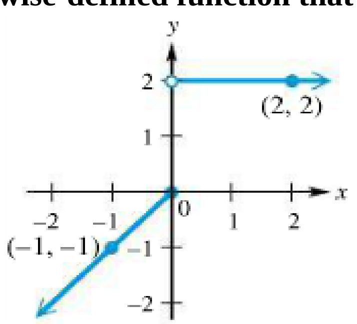


Determine if the following functions are even, odd, or neither.

4. $f(x) = 3x^5 - x^3 + 7x$
5. $f(x) = \sqrt{x^2 + 1}$
6. $f(x) = x^4 - 5x + 2$
7. $f(x) = |5x|$
8. $f(x) = \frac{x^5}{x^4+7}$

Give the equation of the function whose graph is described.

9. The graph of $y = \sqrt[3]{x}$ is shifted 2 units to the left. This graph is then vertically stretched by applying a factor of 1.5. Finally, the graph is shifted 8 units upward.
10. The graph of $y = |x|$ is shifted to the right 3 units. This graph is then reflected about the x-axis. Finally, the graph is shifted 5 units down.
11. **Write a piecewise-defined function that represents the graph below. Then give the domain and range**



12. **Graph the piecewise defined function**

$$f(x) = \begin{cases} 2 & \text{for } x < -1 \\ x^2 & \text{for } x \geq -1 \end{cases}$$

Given $f(x) = x^3$ **and** $g(x) = 2x - 1$, **find the following:**

13. $(f - g)(2)$
14. $(fg)(-1)$
15. $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$
16. $(f \circ g)(-2)$

Given $\boldsymbol{f(x) = \frac{2}{x-1}}$ **and** $\boldsymbol{g(x) = \frac{3}{x}}$ **find the following:**

17. $(f \circ g)(x)$ and the domain of $f \circ g$.

Find the average rate of change for the given function over the given interval.

18. $f(x) = x^2 - 1$ from $x = -2$ to $x = 3$
19. $f(x) = x^2 - 1$ from $x = -2$ to $x = -2 + h$

Find the difference quotient for the following function.

20. $f(x) = 2x^2 - 3x$

Find the equation of the quadratic function satisfying the given conditions. Write your answer in the form $P(x) = ax^2 + bx + c$.

21. Vertex: $(-6, -12)$; through $(6, 24)$
22. Vertex: (2, 5); through (0, 1)

Write the following function in vertex form.

23. $f(x) = 3x^2 + 3x - 6$

Find the vertex of the graph of the following function.

24. $f(x) = 2x^2 - 4x + 5$

Solve.

25. $(4x+1)^2 = 20$
26. $(x+4)(x-1) = -5x-4$
27. $x^2 - 2x = -5$
28. $s = s_0 + gt^2 + k$ for t
29. $x^2 + 4x \leq -3$
30. $x^2 - x + 1 < 0$

Use the function $f(x) = -2x^4 + 2x^3$ to answer the following.

31. Use the leading coefficient test to determine the graph's end behavior.
32. Find the x-intercept(s).
33. Find the y-intercept(s).
34. Determine whether the function is even, odd, or neither.
35. What are the maximum number of turning points?

Divide.

36. $\frac{-x^3-x-5}{x+1}$
37. $\frac{x^7+1}{x+1}$
38. $\frac{3x^4-2x^2-5}{3x^2-5}$

Use synthetic division to find P(k).

39. $P(x) = 5x^3 + 2x^2 - x + 5$, $k = -2$

Find all zeroes of the following function.

40. $P(x) = 3x^3 + 5x^2 - 3x - 2$ given that -2 is a zero

Completely factor the following function.

41. $f(x) = 24x^3 + 40x^2 - 2x - 12$

Find a cubic polynomial with the following zeroes:

42. 4 and $2 + i$

Sketch a rough graph of the following function by finding the end behavior, zeroes, and their multiplicity

43. $f(x) = x^3 + x^2 - 8x - 12$

Solve each equation and inequality.

44. $2x^3 = 4x^2 - 2x$
45. $x^3 - 3x^2 - 6x + 8 = 0$
46. $x^3 - 3x^2 - 6x + 8 > 0$

Find all asymptotes (horizontal, vertical, and slant) of the following functions.

47. $g(x) = \frac{5x^2+4x+3}{5x+9}$

48. $f(x) = \frac{x-2x^2-11}{x^2-3}$

Solve.

49. $\frac{1}{x+2}+\frac{3}{x+7}=\frac{5}{x^2+9x+14}$

50. $\frac{2x-5}{x^2-1}\geq 0$

51. $6-\frac{2}{x+3}=\frac{3x-5}{x+3}$

52. $\frac{1}{x-1}+\frac{1}{x+1}>\frac{3}{4}$

53. $\frac{3}{x+1}<\frac{4}{x+2}$

Determine the domain of each function.

54. $f(x)=-\sqrt[4]{2-0.5x}$

55. $f(x)=\sqrt[5]{x+32}$

56. $f(x)=\sqrt{81-x^2}$

Simplify the following expression so that it does not contain negative exponents

57. $(x+3)^{-\frac{1}{5}}+(x+3)^{-\frac{2}{5}}$

Determine if the following functions are one-to-one. If so, find their inverse. If not, state "not one-to-one"

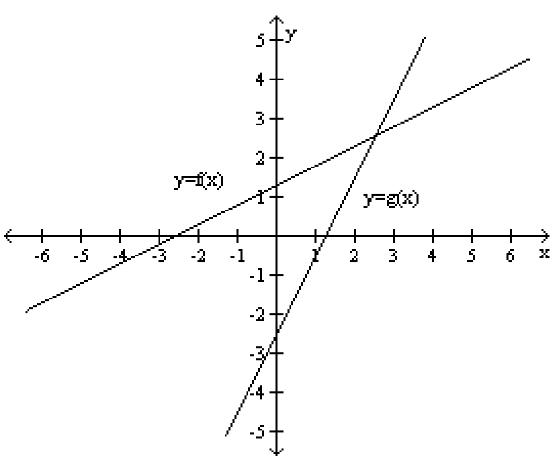
58. $y=-x^2+2$

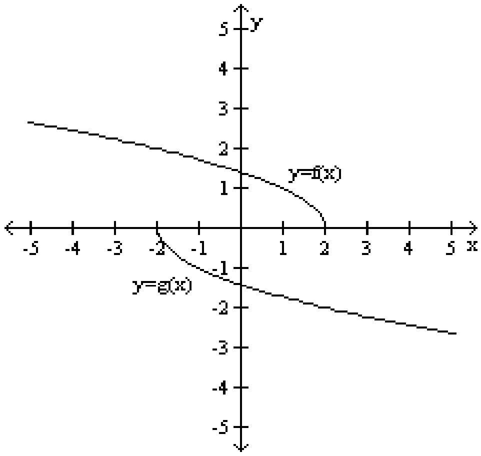
59. $f(x)=-\sqrt{x^2-16},\ x\geq 4$

60. $f(x)=\frac{4-x}{5x}$

61. $f(x)=\frac{2x+1}{x-1}$

Decide whether the pairs of functions are inverses.

62. 

63. 

Use the properties of logarithms to expand the expression completely, if possible

64. $\log_2 \frac{6x}{y}$

65. $\log_6(7m+3q)$

66. $\log_2 \frac{2\sqrt{3}}{5p}$

67. $\log_p \sqrt[3]{\frac{m^5}{kt^2}}$

Use the properties of logarithms to rewrite each expression as a single logarithm with coefficient 1.

68. $\log_b(2y+5)-\frac{1}{2}\log_b(y+3)$

69. $-\frac{2}{3}\log_5 5m^2+\frac{1}{2}\log_5 25m^2$

70. $\frac{4}{3}\ln m-\frac{2}{3}\ln 8n-\ln m^3n^2$

Find the domain of each logarithmic function.

71. $y=\ln(x^4+8)$

72. $y=\log(x^3-81x)$

73. $y=\log\left(\frac{x+1}{x-5}\right)$

74. $y=\log|6x+6|$

Use the change-of-base rule to find an approximation for each logarithm to 3 decimal places.

75. $\log_{200} 175$

76. $\log_{5.8} 12.7$

Solve. Express all solutions in exact form.

77. $5^{2x+1} = 25$
78. $e^{5x-7} = (e^3)^x$
79. $\left(\frac{1}{2}\right)^x = 5$
80. $3^{x-4} = 7^{2x+5}$
81. $\left(\frac{1}{3}\right)^x = -3$
82. $5(1.2)^{3x-2} + 1 = 11$
83. $3^{2x} + 35 = 12(3^x)$
84. $(\log_2 x)^2 + \log_2 x = 2$
85. $\log_5(8-3x) = 3$
86. $\ln x + \ln(x^2) = 3$
87. $\ln(4x-2) - \ln 4 = -\ln(x-2)$
88. $2\log_2(5x-3) + 1 = 17$
89. $\frac{1}{4}e^{2x} + 2e^x = 3$
90. $\log_5(x+2) + \log_5(x-2) = 1$

Solve each formula for the indicated variable.

91. $T = T_0 + (T_1 - T_0)10^{-kt}$, for t
92. $y = \frac{K}{1+ae^{-bx}}$, for b
93. $d = 10\log\left(\frac{I}{I_0}\right)$, for I

Sketch a graph of the following function.

94. $f(x) = \left(\frac{1}{3}\right)^{x+2}$

Solve each system.

95. $\begin{aligned} 4x - 5y &= -11 \\ 2x + y &= 5 \end{aligned}$

96. $\begin{aligned} 2x - 7y &= 8 \\ -3x + \frac{21}{2}y &= 5 \end{aligned}$

97. $$\begin{aligned} 4x - y + 3z &= -2 \\ 3x + 5y - z &= 15 \\ -2x + y + 4z &= 14 \end{aligned}$$

Solve using Matrix Row Operations.

98. $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 8 \\ 2 & 3 & -1 & -2 \\ 3 & -2 & -3 & 15 \end{array}\right]$$

99. $$\left[\begin{array}{ccc|c} 6 & -1 & -1 & 4 \\ -12 & 2 & 2 & -8 \\ 5 & 1 & -1 & 3 \end{array}\right]$$

Graph the system of Linear Inequalities.

100. $$\begin{aligned} y &\geq x - 4 \\ x + y &\leq 2 \end{aligned}$$

101. $$\begin{aligned} x + y &< 1 \\ y &> x - 1 \end{aligned}$$

Solve each linear programing problem.

102. Minimize $z = 2x + 5y$, with constraints $x \geq 0,\ y \geq 0,\ x + y \geq 2,\ x \leq 5,\ y \leq 3$.
103. Maximize $z = 5x + 3y$, with constraints $x \geq 0,\ y \geq 0,\ x + y \geq 2,\ x + y \leq 8,\ 2x + y \leq 10$.

Applications.

104. The student activities department of a college plans to rent buses and vans for a special trip. Each bus has 40 regular seats and 1 special seat for travelers with disabilities. Each van has 8 regular seats and 3 special seats. The rental for each van is $350 and the rental cost for each bus is $975. If 320 regular seats and 36 special seats are required for the trip, how many of each type of vehicle should be rented to minimize cost?

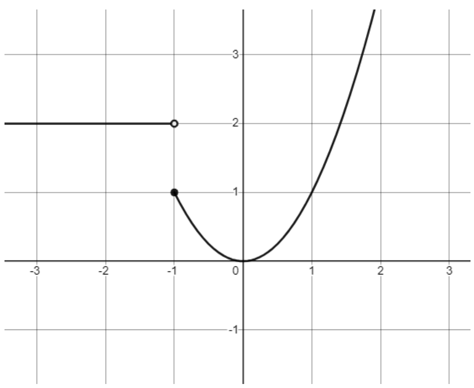
105. A baseball is hit so that its height in feet after t seconds is $s(t) = -16t^2 + 44t + 4$. How high is the baseball after 1 second? Find the maximum height of the baseball.

106. A raised wooden walkway is being constructed through a wetland. The walkway will have the shape of a right triangle with one leg 700 yards longer than the other and the hypotenuse 100 yards longer than the longer leg. Find the total length of the walkway.

107. You have 80 yards of fencing to enclose a rectangular region. Find the dimensions of the rectangle that maximize the enclosed area. What is the maximum area?

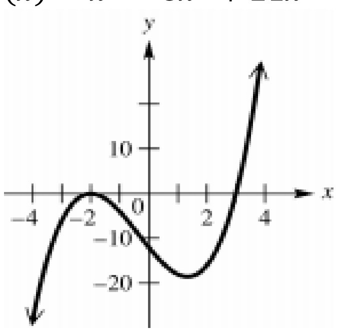
108. The growth of bacteria in food products makes it necessary to date some products (such as milk) so that they will be sold and consumed before the bacterial count becomes too high. Suppose that, under certain storage conditions, the number of bacteria present in a product is $f(t) = 500e^{0.1t}$, where t is time in days after packing of the product and the value of $f(t)$ is in millions. If the product cannot be safely eaten after the bacterial count reaches 3,000,000,000, how long will this take?

Answer Key

1. $y = \frac{1}{5}x + \frac{1}{10}$
2. $y = \frac{3}{5}x - 7$
3. (a) none (b) $(-\infty, -2)$; $(3, \infty)$ (c) $(-2, 3)$ (d) $(-\infty, \infty)$ (e) $(-\infty, 1.5] \cup [2, \infty)$
4. Odd
5. Even
6. Neither
7. Even
8. Odd
9. $y = 1.5\sqrt[3]{x+2} + 8$
10. $y = -|x-3| - 5$
11. $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$ Domain: $(-\infty, \infty)$ Range: $(-\infty, 0] \cup \{2\}$
12. 

13. 5
14. 3
15. Undefined
16. -125
17. $(f \circ g)(x) = \frac{2x}{3-x}$, Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$
18. 1
19. $h - 4$
20. $4x + 2h - 3$
21. $P(x) = \frac{1}{4}x^2 + 3x - 3$
22. $P(x) = -x^2 + 4x + 1$
23. $f(x) = 3\left(x + \frac{1}{2}\right)^2 - \frac{27}{4}$
24. $(1, 3)$
25. $\left\{\frac{-1 \pm 2\sqrt{5}}{4}\right\}$

26. $\{-8, 0\}$
27. $\{1 \pm 2i\}$
28. $t = \dfrac{\pm\sqrt{(s - s_0 - k)g}}{g}$
29. $[-3, -1]$
30. $\emptyset$
31. As x approaches $\pm\infty$, $f(x)$ approaches $-\infty$
32. $(0,0)$ and $(1,0)$
33. $(0,0)$
34. Neither
35. 3
36. $-x^2 + x - 2 + \frac{-3}{x+1}$
37. $x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$
38. $x^2 + 1$
39. -25
40. $x = -2, \frac{1 \pm \sqrt{13}}{6}$
41. $f(x) = 2(2x+3)(3x+2)(2x-1)$
42. $P(x) = x^3 - 8x^2 + 21x - 20$
43.



44. $x = 0,1$
45. $x = -2,1,4$
46. $(-2,1) \cup (4, \infty)$
47. VA: $x = -\dfrac{9}{5}$ HA: None SA: $y = x - 1$
48. VA: $x = \sqrt{3},\ x = -\sqrt{3}$ HA: $y = -2$ SA: None
49. $\emptyset$
50. $(-1,1) \cup \left[\frac{5}{2}, \infty\right)$
51. $x = -7$
52. $\left(-1, -\dfrac{1}{3}\right) \cup (1,3)$
53. $(-2,-1) \cup (2, \infty)$
54. $(-\infty, 4]$

55. $(-\infty,\infty)$
56. $[-9,9]$
57. $\frac{(x+3)^{\frac{1}{5}}+1}{(x+3)^{\frac{2}{5}}}$
58. $x=4$
59. $x=-1,3$
60. $\{16\}$
61. $\left\{\frac{1}{5},\frac{3}{4}\right\}$
62. Yes
63. No
64. $\log_2 6+\log_2 x-\log_2 y$
65. Cannot be rewritten
66. $1+\frac{1}{2}\log_2 3-\log_2 5-\log_2 p$
67. $\frac{1}{3}(5\log_p m-\log_p k-2\log_p t)$
68. $\log_b \frac{2y+5}{\sqrt{y+3}}$
69. $\log_5(5^{1/3}m^{-1/3})$, or $\log_5\sqrt[3]{\frac{5}{m}}$
70. $\ln\sqrt[3]{\frac{1}{64m^5n^8}}$
71. $(-\infty,\infty)$
72. $(-9,0)\cup(9,\infty)$
73. $(-\infty,-1)\cup(5,\infty)$
74. $(-\infty,-1)\cup(-1,\infty)$
75. 0.975
76. 1.446
77. $x=\frac{1}{2}$
78. $x=\frac{7}{2}$
79. $x=\frac{\log 5}{\log\left(\frac{1}{2}\right)}$
80. $\left\{\frac{5\log 7+4\log 3}{\log 3-2\log 7}\right\}$
81. No solution
82. $\frac{2}{3}+\frac{\log 2}{3\log 1.2}$
83. $\log_3 5, \log_3 7$

84. $x = \frac{1}{4}, 2$

85. $\{-39\}$

86. e

87. 2.5

88. $\left\{\frac{259}{5}\right\}$

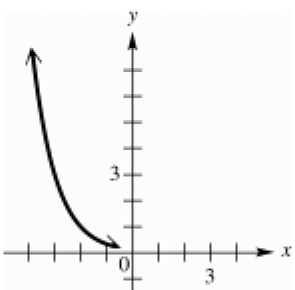
89. $\left\{\ln\left(2\sqrt{7}-4\right)\right\}$

90. 3

91. $t = -\frac{1}{k}\log\left(\frac{T-T_0}{T_1-T_0}\right)$

92. $b = \frac{\ln\left(\frac{K-y}{ay}\right)}{-x}$

93. $I = I_0 \cdot 10^{d/10}$

94. 

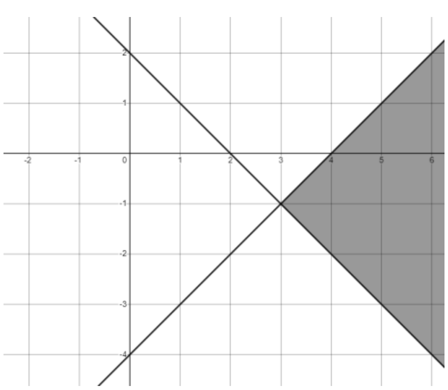
95. $\{(1,3)\}$

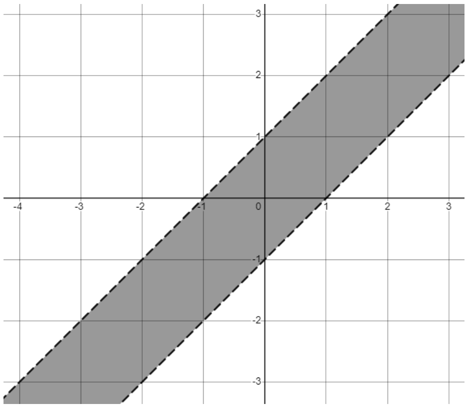
96. No solution

97. $\{(-1,4,2)\}$

98. $\{(4,-3,1)\}$

99. $\left\{\begin{matrix} x = \frac{2}{11}z + \frac{7}{11} \\ y = \frac{1}{11}z - \frac{2}{11} \end{matrix}\right\}$ where z can be any real number

100. 

101. 
102. Minimum of $z = 4$ at $(2,0)$
103. Maximum of $z = 28$ at $(2, 6)$
104. 6 buses and 10 vans
105. 32 ft, 34.25 ft
106. 3000 yards
107. 20 yards by 20 yards; 400 yards squared
108. 18 days